Modelling War Phenomena with an emphasis on Operation Research

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***Abstract*—Different domains of mathematics and informatics have a great impact on a wide range of army needs. Mathematical models when formulated with appropriate parameters can determine an approximate state of war progress. Fields of operations research are of interest to military scientists primarily to recreate war phenomena inventory maintained, using simulations. This paper reviews basic war models such as Lanchester and Guerrilla models and proposes an inventory model with a practical simulation to demonstrate its applicability in modelling food inventory.**

Keywords—Differential Equations, Inventory Models, Operations Research, War models.

I. Introduction

Warfare encompasses three broad areas of knowledge. The first being human behavior which goes into fields of psychology to study how humans react in different situations. Second, being physics which focusses on the mechanics of performance of machines. The third field is the environment in which men and machines often clash with each other [1]. As a consequence, the warfare field lies in the intersection of mathematical modelling and military science. It delves into fields like war predictions through models which is the main topic of discussion in this paper. Some fields of mathematics that are of interest to military scientists are:

A. Applied Analysis

It deals with aspects such as mathematical modelling of recurrence relations, differential equations (ordinary or partial), and integral equations for advanced materials, fluid flow and nonlinear dynamics.

B. Computational Mathematics

The evolution of the army into a modern, technology-based force places has increasing demands on numerical methods and optimization for faster, more stable, and accurate solutions to problems in the physical sciences. Since it may not be always possible to solve a differential equation analytically to find it's exact solution, this is where numerical methods come into the picture.

C. Probability and Statistics

This is one of the most important fields as a lot of the models built revolve around data. With data comes distributions and with distributions come techniques for estimating parameters for them. Once the parameters are estimated, techniques that help evaluate the *goodness of fit* help us evaluate several reference models.

II. Related Work

War models such as Lanchester and Guerrilla [8] have been implemented in several combat models [9] and have shown success in modelling historical battles [10, 11]. These models are inspired by biological phenomena [12] (predator-prey) with application extending to domains such as video gaming [13] and advertising [14]. Variations of these models have attempted to incorporate landscape effects using partial differential equations [2] and irregular warfare [15]. In this study, we highlight some properties that models should encapsulate based on a case study.

Inventory management [16, 17] has been dominantly used by manufacturing companies to model material flow and purchased inventory items. Different variations of the vanilla model: economic order quantity (EOQ) like ABC [18] and JIT (Just in Time) [19] incorporate uncertainty at different stages of the procurement pipeline differently. To the best of our knowledge, inventory management in the context of modelling food supplies for the military has not been explored much.

The main contributions of this paper can be summarized as follows:

- In Section III, we mention some of the common war models. The limitations of these models are highlighted through a case study and towards the end, we highlight the factors that must be taken into consideration.
- In Section IV, we demonstrate the applicability of the vanilla inventory model in the military context for modelling food supplies to minimize costs incurred.

III. War Models

A. Lanchester Model

Lanchester (*1916*) modelled a war between two parties with the help of a pair of linear ordinary differential equations. According to this model, the rate of decrease in members of an army is directly proportional to the number of people present in the opposition army. Equation (1) and (2) describe the model.

$$\frac{dx}{dt} = -b.y \tag{1}$$

$$\frac{dy}{dt} = -a.x \tag{2}$$

Where x and y represent the number of soldiers in the first and second army respectively. The soldiers in both the armies at the start are given by x_0 and y_0. Army efficiency is represented by a and b respectively. Equation (3) describes the condition when the first army wins and (4) describes the condition when the second army wins.

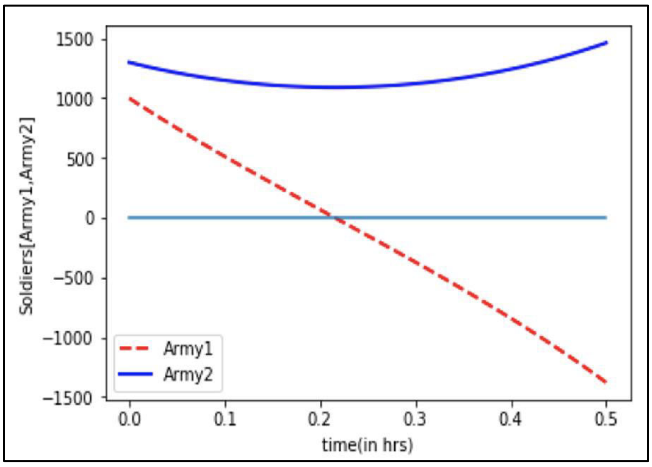

Fig. 1. Army population as a function of time

$$x_0^2.b > y_0^2.a \tag{3}$$

$$x_0^2.b < y_0^2.a \tag{4}$$

Fig. 1. shows how the number of people on either side as a function of time with parameters stated below. The initial conditions are considered as below.

Army 1 efficiency - *2* kills/hour
Army 2 efficiency - 4 kills/hour
Army 1 initial soldiers - 1000
Army 2 initial soldiers – 1300

B. Guerrilla War Model

This kind of war model is used when either one army or both the army parties are hiding [4]. In the case where both the armies are hiding, the factors that contribute to the killing of soldiers on either side are as follows:

- The number of soldiers on their side, as more the number of soldiers greater is the probability of them being found.
- The number of soldiers on the opposite side, as when they are more in number, they can find the opposite party easily

In the case where a single army is hiding, the decay rate of the army that is hiding would remain unchanged, while that of the open one is proportional to the number of soldiers in the other army. Assuming that there is a red army (R) and a blue army (B), (5) and (6) effectively model the aforementioned situation.

$$\frac{dB}{dt} = (-\beta)BR \tag{5}$$

$$\frac{dR}{dt} = (-\gamma)BR \tag{6}$$

R(t) and *B(t)* denote the number of soldiers in the red and blue army respectively at time t. The efficiency of the red and blue armies is given by β and γ, respectively. *R(0)* and *B(0)* are the initial populations of respective armies. Equation (7) and (8)

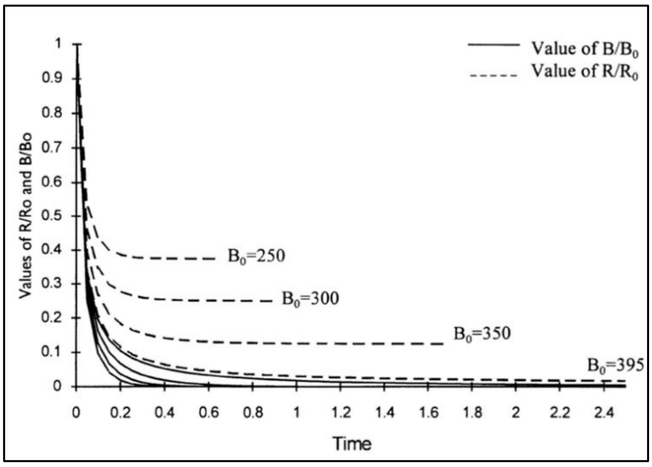

Fig. 2. R/R(0) and B/B(0) as a function of time

describe the conditions when Red or Blue army will win respectively.

$$\beta.R(0) > \gamma.B(0) \tag{7}$$

$$\beta.R(0) < \gamma.B(0) \tag{8}$$

Fig. 2. illustrates the application of the above model in the case where both armies are hiding with *R(0)* = 200, β = 0.2 and γ = 0.1

C. Case Study

In this section, we demonstrate the limitations of the Lanchester model by estimating model parameters (a, b) using World War II (WWII) from [7]. The optimal army efficiencies were found by carrying out a grid search in $[0.001, 1]$ X $[0.001,1]$ with a width of 0.01 along each direction. The sum of squares errors was minimized to arrive at Soviet Army efficiency (a) of 0.01 and German Army efficiency (b) of 0.102. *Fig. 3.* shows how the model solution compares with the actual data. The reason for the gap between the estimated actual solution is mainly due to the model considering very limited factors (i.e., population of armies only) and time-independent model parameters.

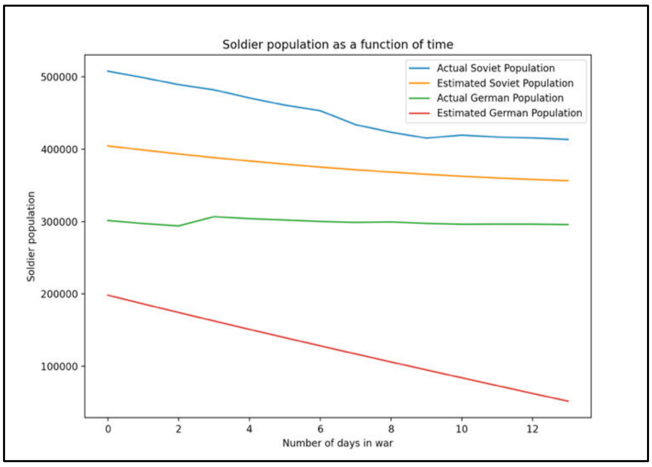

Fig. 3. Soviet and German army population as a function of time

D. *Desirable Properties of models*

Here we highlight some of the properties that should be kept in mind during the design of war models.

- Models must exhibit saturating behavior in the absence of external factors as time progresses. As it can be seen that it makes sense to consider Lanchester model in a truncated domain where the population is positive.
- They should encapsulate information related to the current state of war, for example, both the aforementioned models assume constant efficiency rates, which would ideally not be the case as we would expect efficiency to be impacted by time, army population as well as external factors.
- Apart from temporal considerations, wars also influenced by the terrain landscape (such as hill, rivers) where the war happens. To consider such effects into consideration, one can build models using PDEs as discussed in [2,3].

IV. Inventory Models

These are mathematical models [5,6] that help organizations determine the optimum level of inventories to be maintained in production processes. Here we focus on finding out the efficient amount of inventory level that should be maintained by the army to minimize their cost.

Such work has application in modelling food supplies that need to be maintained by the army. If fewer supplies are maintained, then it would be enough to meet requirements and if too much is maintained, it would be difficult to move around in hostile conditions. Thus, it is very important to maintain the correct amount of supplies.

A. *Model Setting*

- The study proposes a model for one person and then accordingly finds the result for a group of people by multiplying the appropriate factor.
- It assumes that the probability distribution of consumption of food is Gaussian with a mean of 2.5 kilos and a standard deviation of 0.5 kilos.
- It assumes purchase costs (*PC*), ordering costs (*OC*) and holding cost (*HC*) to be Rs. 200/kg, Rs 160/order, Rs 30/kg, respectively.
- It assumes the reordering point (the point at which order is placed) as 4 kg as the probability of a person consuming more than 4kg is very low.
- Lead time is assumed to be 0 (i.e., if the army places an order for food supplies, it will get the order the next day morning).

W	Week number
OR	Units received in the beginning of the week
OI	Opening Inventory for the week
Rnd	Random number for demand variable
D	Shows demand for the week
EI	Ending inventory

Opening inventory (*OI*) is assumed to be *6*. Simulation was carried out for *50* weeks. Results of the simulation for first 6 weeks are shown in Table I.

TABLE I. Inventory Model Simulation for Six Weeks

W	*OR*	*OI*	*Rnd*	*D*	*EI*	*OP*	*OC*	*PC*	*HC*
1	0	6.0	0.33	2.5	3.5	0	0	0	105
2	0	3.5	0.51	2.5	1.0	4	160	800	30
3	4	5.0	0.61	2.5	2.5	4	160	800	75
4	4	6.5	0.51	2.5	4.0	0	0	0	120
5	0	4.0	0.12	2.0	2.0	4	160	800	60
6	4	6.0	0.92	3.0	3.0	0	0	0	90

B. *Model observations*

- The total cost for 50 weeks is Rs. 32295
- Cost per week is Rs. 649.5
- By performing iterations over different opening inventory values between 1.0 and 6.0, the optimal value for opening inventory was found at 3.27 kg with the optimal cost of Rs. 566.07.

Fig. 4. shows how the cost per week varies as a function of opening inventory.

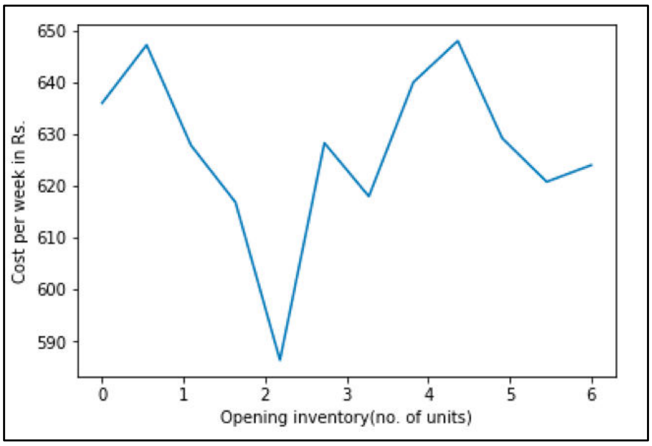

Fig. 4. Cost per week as a function of Opening Inventory

V. CONCLUSION

In the beginning, the study explores simple war models and lays out the factors that must be taken into consideration. With the advent of latest technologies, there are several factors apart from those present on the battlefield which influence the course of warfare, such as intercepting communication channels. The work also proposes an inventory model which can be effectively used to compute optimal stock values to be maintained for a predefined set of parameters. The generalized model can be used as a basis for the construction of more complex models which take into case-specific features.

REFERENCES

[1] D. C Arney, J. Myers. "Mathematical Research and Modelling for the Army"

[2] V. Protopopescu, R. Santoro, and J. Dockery. "Combat modeling with partial differential equations". *European Journal of Operations Research*, vol. 38, no. 2, pp. 178–183, 1989

[3] Keane, Therese. "Combat Modelling with Partial Differential Equations." *Applied Mathematical Modelling*, vol. 35, no. 6, pp. 2723–35, 2011

[4] Deitchman, S. J. "A Lanchester Model of Guerrilla Warfare." *Operations Research*, vol. 10, no. 6, pp. 818–827, 2011.

[5] Daras, Nicholas. Applications of Mathematics and Informatics in Military Science (Springer Optimization and Its Applications, 71). 2012th ed., Springer, 2012.

[6] Jaiswal, N. Military Operations Research: Quantitative Decision Making (International Series in Operations Research & Management Science, 5). Softcover reprint of the original 1st ed. 1997, Springer, 2012.

[7] Lucas, Thomas W., and Turker Turkes. "Fitting Lanchester Equations to the Battles of Kursk and Ardennes." *Naval Research Logistics (NRL)*, vol. 51, no. 1, pp. 95–116, 2003.

[8] Lanchester, F.W. Aircraft in Warfare: The Dawn of the Fourth Arm; Constable and Co.: London, UK, 1916

[9] Brown, G.; Washburn, A. The Fast Theater Model (FATHM). Mil. Oper. Res., vol. 12, no. 4, pp. 33–45, 2007.

[10] Sahni, M.; Das, S.K. Performance of maximum likelihood estimator for fitting Lanchester equations on Kursk Battle data. J. Battlef. Technol., vol. 18, no. 2, pp. 23–30, 2015.

[11] MacKay, N.J.; Price, C.; Wood, A.J. Weight of Shell Must Tell: A Lanchestrian reappraisal of the Battle of Jutland. History, vol. 101, pp. 536–563, 2016.

[12] Eldrige, S.A.; Mesterton-Gibbons, M. Lanchester's attrition models and fights among social animals. Behav. Ecol., vol. 14, pp. 719–723, 2003

[13] Johnson, D.D.P.; MacKay, N.J. Fight the Power: Lanchester's laws of combat in human evolution. Evol. Hum. Behav., vol. 36, pp. 152–163, 2013

[14] Stanescu, M.; Barriga, N.; Buro, M. Using Lanchester Attrition Laws for Combat Prediction in StarCraft. In Proceedings of the Eleventh AAAI Conference on Artificial Intelligence and Interactive Digital Entertainment (AIIDE-15), Santa Cruz, CA, USA, pp. 86-92, 2015

[15] Kress M. Lanchester Models for Irregular Warfare. *Mathematics*., vol. 8 no. 5:737

[16] I. Ahmed, and I. Sultana, "A literature review on inventory modeling with reliability considerations," International Journal of Industrial Engineering Computations, vol.5, no. 1, pp.169-178, 2014.

[17] S. Ziukov, "A literature review on models of inventory management under uncertainity", Business system and Economics, vol.5, no.1, 2015

[18] R. S. Angel, S. N. Gomathi, and G. Chitra, "Inventory Management – A case study," International Journal of Emerging Research in Management & Technology, vol.3, no.3, 2014.

[19] K. Rahmani and M. A. Nayebi, "Effect of JIT implementation inIran Automotive industry :Case study: Iran Khodro's Assembly line 2)," Indian Journal of Science Research, vol. 7, no.1, pp. 001-016, 2014